BLOCK CODES

INVOLVES A GENERATOR MATRIX ($\underline{G}$)

BLOCK IS CALLED THE MESSAGE ($\underline{m}$)

ENCODER OUTPUT IS CALLED THE CODEWORD ($\underline{c}$)

$$\underline{c} = \underline{m}\,\underline{G}$$

DEFINITIONS

A SYSTEMATIC BLOCK CODE MEANS THAT THE MESSAGE IS IN THE CODEWORD

MESSAGE 1011010 → CODEWORD 1011010.......

+ MORE BITS

IF IT IS NOT A SYSTEMATIC CODE THEN EVEN THOUGH THE MESSAGE IS STILL IN THE CODEWORD IT IS SCRAMBLED AND WOULD NEED "SPECIAL DECODING"

FOR SYSTEMATIC ENCODERS THE GENERATOR MATRIX MUST CONTAIN AN IDENTITY SUB-MATRIX

BLOCK CODES ARE LINEAR BECAUSE THE GENERATOR MATRIX IS MADE UP OF CODEWORDS

HAMMING DISTANCE IS THE NUMBER OF BINARY DIGITS THAT DIFFER BETWEEN ANY TWO CODE WORDS

HAMMING WEIGHT IS THE NUMBER OF ONES (1'S) IN A CODEWORD

HAMMING DISTANCE BETWEEN ANY TWO CODEWORDS IS EQUAL TO THE WEIGHT OF THEIR SUM (XOR)

MODULO-2 ADDITION

$$\underline{G} = [I_k \; P]$$

$$\underline{H} = [P^T \; I_{n-k}]$$

$$G = \left[\; \overset{7}{\longleftrightarrow} \; \updownarrow 4 \;\right] \qquad n = 7, \; k = 4$$

$$\underline{G} \, \underline{H}^T = 0$$

7,4 CODE

CODEWORD LENGTH

MESSAGE LENGTH

$\underline{C} \longrightarrow \underline{Y}$ NOISY WORD

TRANSMITTED

RECEIVED CODE WORD WITH ERRORS

$\underline{Y} = \underline{C} + \underline{e}$ WHERE $\underline{e}$ IS THE ERROR VECTOR

$$\underline{C} = \underline{m}\,\underline{G}$$

$$\underline{C}\,\underline{H}^T = \underline{m}\underbrace{(\underline{G}\,\underline{H}^T)}_{0} = \underline{m}\,\underline{0} = 0$$

← ONLY OCCURS FOR $\underline{C}$ THE CORRECT CODEWORD WITH <u>NO ERRORS</u>

$\underline{Y}\ \underline{H}^T = \underline{S}$ (SYNDROME)

LOWER CASE R

ZERO SYNDROME MEANS NO ERRORS

NON-ZERO SYNDROME WOULD BE OF THE FORM [...... 0100]

WE CAN RECOGNIZE IF OUR BLOCK CODE IS A HAMMING CODE BY THE FOLLOWING BEING TRUE

$n = 2^m - 1$ (WHERE m IS AN INTEGER)

$k = 2^m - 1 - m$

7,4 CODE
15,11 CODE } HAMMING CODES

$$\tau(\text{ERROR}) \leq \left\lfloor \frac{d_{mm}-1}{2} \right\rfloor \rightarrow \text{LOWEST INTEGER}$$

LOWEST HAMMING DISTANCE IS d_{mm}

$\therefore$ IF YOUR LOWEST HAMMING DISTANCE WITHIN THE CODE UNDER CONSIDERATION IS 3

ACCORDING TO OUR FORMULA YOU CAN ONLY CORRECT ONE ERROR

4 BITS MESSAGE

16 POSSIBLE COMBINATIONS OF BITS GOING INTO OUR ENCODER

0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

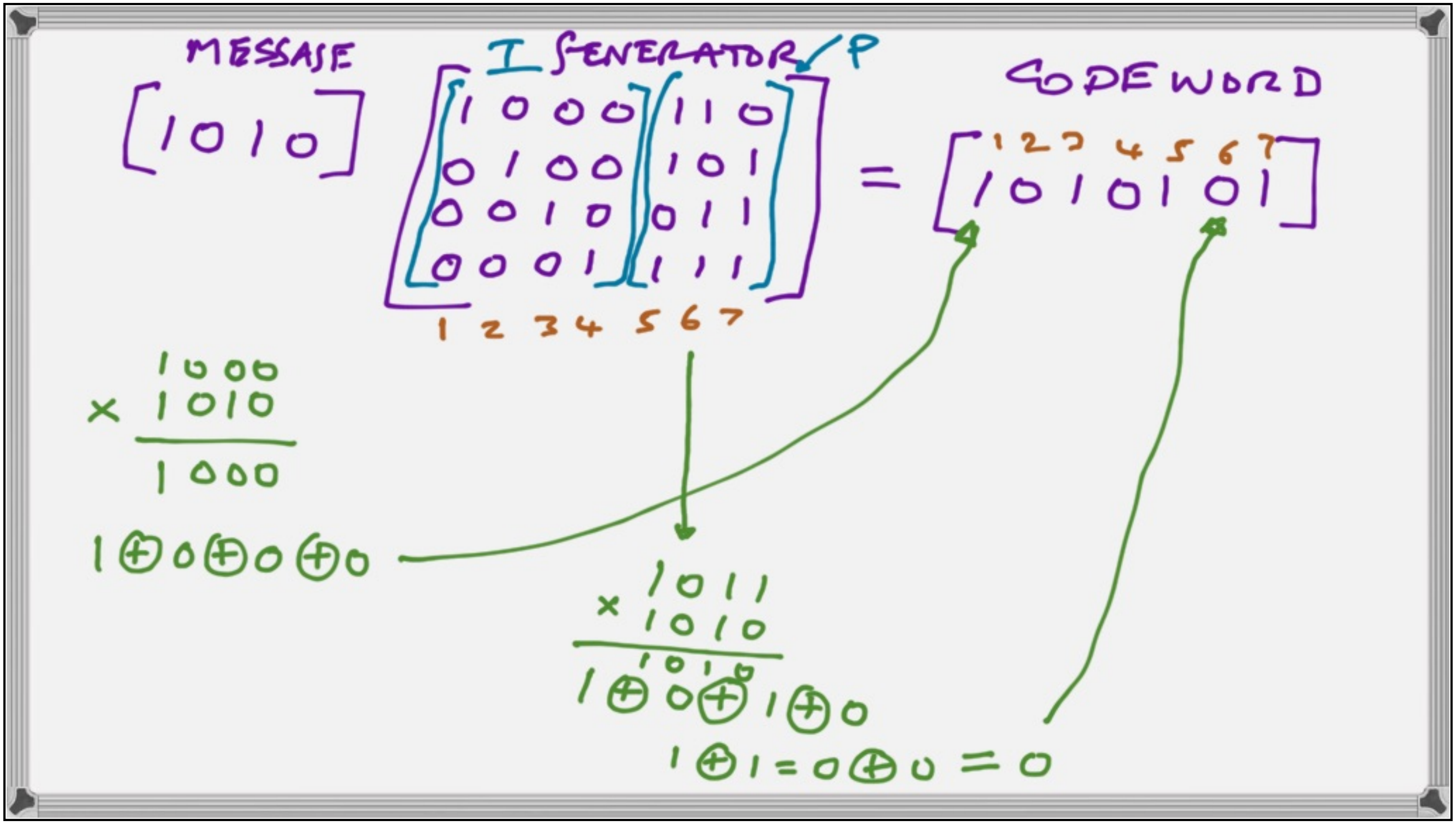
$2^4 = 16$

CODEWORD 7 BITS

WE ARE ONLY USING 16 CODEWORDS OUT OF A TOTAL OF 128 POSSIBLE CHOICES

IS WHAT IS RESPONSIBLE FOR THE "ERROR CORRECTING CAPABILITY"

$2^7 = 128$


MESSAGE
[1 0 1 0]
I GENERATOR P
1 0 0 0 1 1 0
0 1 0 0 1 0 1
0 0 1 0 0 1 1
0 0 0 1 1 1 1
1 2 3 4 5 6 7
=
CODEWORD
1 2 3 4 5 6 7
1 0 1 0 1 0 1
1000
× 1010
1000
1 ⊕ 0 ⊕ 0 ⊕ 0
1011
× 1010
1010
1 ⊕ 0 ⊕ 1 ⊕ 0
1 ⊕ 1 = 0 ⊕ 0 = 0

$$\underline{H} = \left[P^T \; I_{n-k} \right] \qquad n=7, \; k=4$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = P$$

$$7-4$$

$$\underline{H} = \left[P^T \; I_3 \right]$$

$$P^T = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

$$\underline{H} = \left[\begin{array}{cc} P^T & I_3 \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \right]$$

$$\underline{H}^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

IDENTITY MATRIX HAS A DIAGONAL OF ALL "1's" AND THE REST ARE ZEROES SO THE MOST IMPORTANT QUESTION IS "HOW BIG IS IT?"

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

WHAT IS IT'S PURPOSE?

$$\begin{bmatrix} 7 & 4 & 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 4 & 3 & 2 \end{bmatrix}$$

THIS IS HOW WE GET THE MESSAGE BACK INTO THE CODEWORD